The Vortex Frequencies of Bluff Cylinders at Low Reynolds Numbers

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Experiments on the vortex shedding frequencies of selected bluff cylinders were conducted in the 1½ ft by 4 ft (0.46 m by 1.2 m) test section of the Maryland University wind tunnel. The resulting data show how the Strouhal number and dimensionless frequency of an elliptical cylinder vary with flow incidence angle in the Reynolds-number range between 300 and 1200. These results are discussed in the light of the results of other investigations. Similar data are also presented for a square cylinder in the same range of Reynolds numbers. It is shown that the data for elliptical cylinders with eccentricities between 0.6 and 0.8 can be correlated over the range of Reynolds numbers between 500 and 10^5. A correlation of data on Strouhal-number variation with angle of incidence, based on the projected height of the model, is obtained for both the square and elliptical models in the range of Reynolds numbers between 300 and 1200. The validity of the Independence Principle for flow past a yawed circular cylinder is examined. Experimental data, based on the velocity component normal to the cylinder, are presented for flow past a circular cylinder yawed 40 deg. These data are compared with those of other investigators. The combined data, which cover a wide range of Reynolds numbers, show a definite departure from the Independence Principle.

Nomenclature

a	= semimajor diameter of an ellipse
b	= semiminor diameter of an ellipse
C_D	= pressure-drag coefficient, (pressure drag/unit span)/($\frac{1}{2}\rho U^2 d$)
d	= projected cylinder height on a plane normal to the flow direction
d_0	= projected height of a cylinder at zero-degrees incidence to flow
d_s	= lateral dimension of a square
d_t	= transverse dimension of a rectangular section or diameter of circular section
e	= eccentricity of an ellipse $(1-b^2/a^2)^{1/2}$
f	= frequency with which vortices are shed from one side of cylinder
F	= dimensionless shedding frequency fd^2/ν
Re_d	= freestream Reynolds number Ud/ν
Re_n	= Reynolds number based on velocity U_n
S, S_d, S_n	= Strouhal numbers, fd_t/U, fd/U, and fd/U_n, respectively
U	= freestream velocity
U_n	= component of the freestream velocity that is normal to the cylinder axis
α	= angle of incidence to flow direction
θ	= yaw angle
ν	= kinematic viscosity

Subscripts

n	= component normal to cylinder axis
0	= based on a flow incidence angle of zero degrees

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Introduction

THE importance of the stationary unyawed cylinder in the study of the aeroelastic instability of bluff cylinders has been brought out by many workers in this field. For the purposes of this study, it will suffice to mention a few brief examples. Some fundamental aspects of aeroelastic instability have been cited by Marris[1] and Parkinson and Brooks.[2] In particular, they indicate that the amplitude and frequency of aerodynamically induced vibrations transverse to the flow direction depend on whether or not a condition of resonance exists between the natural frequency of the elastic system and the Strouhal frequency of the stationary cylinder. Parkinson's work dealt with the motion of spring-mounted rectangular cylinders which had one face normal to the flow and transverse-over-streamwise dimension ratios between 0.375 and 2.0. He found that these sections, as well as circular sections, initiated self-excited transverse vibrations when

$$V \simeq 1/(2\pi S) \tag{1}$$

where $S = fd_t/U$ is the stationary-cylinder Strouhal number, $V = U/nd_t$ is the dimensionless freestream velocity, n is the natural frequency of the elastic system (in radians per second), and d_t is either the transverse dimension of the rectangular section or the diameter of the circular section. The Strouhal-number data used in this analysis included square cylinders and were obtained at Reynolds numbers (based on the transverse dimension) of 33000 and 66000.

More recently, Vickery[3] investigated the Strouhal-number characteristics of a high-aspect-ratio square cylinder at different angles of incidence. These measurements were made at Reynolds numbers between 4×10^4 and 1.6×10^5. Excellent reviews of unsteady aerodynamic forces on bluff structures due to aerodynamic instability are provided by Scruton and Rogers[4] and Parkinson.[5] These discussions, which include both vortex excitation and galloping-type excitation, are primarily concerned with the response of cylinders of either circular or square cross section. Moreover, Hunt[6] describes many recent investigations of the speed and direction of the wind around rectangular structures. Specific consideration is given to the factors influencing the reattachment of the separated flow at certain angles of attack. In a very recent paper, Wilkinson et al.[7] present new information on chordwise and spanwise correlations observed along the side faces of square cylinders which were rigidly positioned with one face normal to the flow. These experiments, performed in either air or water, covered the Reynolds-number range between 10^4 and 10^5. An investigation of the bandwidth of "locking-on" of oscillating square cylinders was also made for $10^3 < Re < 10^4$.

In addition to the rectangular cylinders and circular cylinders, elliptical cylinders of arbitrary eccentricity constitute another class of shapes which are of great importance in the study of aeroelastic instability. The stationary-cylinder Strouhal numbers associated with such geometries can have important application in predicting the vibrational characteristics of fluid probes. In connection with the investigation of the vibrational characteristics of Pitot tubes of noncircular cross section, Marris[8] found that the proximity of the Strouhal frequency of a stationary cylinder to its natural frequency is the factor which determines the cylinder's vibrational characteristics. The Reynolds numbers of these experiments, when based on the major diameter of the oval section, were between 1000 and 2000. A similar result was also obtained by Winternitz[9] for the same Reynolds-number range.

Strouhal-number data for elliptical cross sections which have a moderate degree of eccentricity ($0.6 < e < 0.8$) have not been obtained over the whole subcritical range of Reynolds numbers. Wiland[10] has obtained some data for two ellipses ($e = 0.6$ and 0.8) in the limited range of Reynolds numbers from 3×10^4 to 10^5. Chiu and Lienhard[11] also present data on an elliptical cylinder ($e = 0.865$) at three intermediate Reynolds numbers, which are roughly in the range 8×10^3 to 1.4×10^4. However, data only exist for zero incidence. The only other work on the elliptical cylinder is that due to Delany and Sorenson[12] who tested their model ($e = 0.866$) in the range 2.4×10^5 to 2×10^6. This included zero incidence at the lower (subcritical) Reynolds numbers and 90-deg incidence at the higher (supercritical) Reynolds numbers. From these observations, as well as earlier ones, it appears that there is a complete lack of Strouhal data for stationary rectangular and elliptical cylinders at different angles of incidence.

A related area of interest bearing on Strouhal frequency is the prediction of the form drag of variously shaped cylinders at different angles of incidence. A basis for such estimates is suggested by the drag concept outlined by Roshko[13] and Gerrard.[14] Briefly, it consists of the postulation that two factors determine shedding frequency. One factor is the physical extent of that portion of the wake in which the vortices are formed; this region is termed the formation region. The other factor is the width of the shear layer near the end of the formation region. Only the size of the formation region appears to be important in the determination of form drag.

According to the authors just cited, the size of the formation region behind a given profile depends on the frontal height of the model (obtained by a projection parallel to the flow direction) rather than on any fixed dimension of the profile. In comparing the form drag of different geometries at arbitrary angles of flow incidence, the definition of the Strouhal number must, therefore, be based on this frontal height. According to Gerrard, the shedding frequency increases as the size of the formation region decreases. Since the size of the formation region varies directly with the frontal height, an increase in this height will produce a lower shedding frequency for a given freestream velocity. This assumes that the separation points are not affected by Reynolds-number changes; according to the data presented by Wiland,[10] such an assumption is valid for Reynolds numbers that remain below the critical value. Moreover, since $S_d = fd/U$, the product fd can be nearly the same for similar geometries at the same angle of incidence and at the same U. However, this situation can only exist when the Strouhal number has a very weak dependence on Reynolds number (which is usually the case for fully turbulent wakes). If, on the other hand, one considers different shapes having the same projected height d normal to a flow with velocity U, f will be greater for the smaller formation region (lower form drag). This would lead one to suspect that the drag coefficient might be inversely dependent on the Strouhal number. This, in fact, can be shown by compiling Strouhal and drag data on flows past a variety of bluff profiles. In a compilation made by Hoerner[15], it was found that the data were correlated by the expression

$$S = 0.21 / C_D^{\,2/3} \tag{2}$$

A similar compilation, which was made in the course of the present study, indicates that the ¾ power improves the fit given by Eq. (2).

In view of the preceding considerations, two high-aspect-ratio bluff cylinders, viz., square and elliptical, were selected for the determination of their vortex-frequency characteristics over a wide range of incidence angles and at Reynolds numbers between 300 and 1200. These data would then provide a means for determining the relative magnitudes of the form drag of not only these geometries but also other geometries for which the Strouhal frequencies are known in this Reynolds-number range. At this point it should be noted that there is a basic difference in the two shapes selected. The square cylinder represents a class of shapes having fixed and stable separation points which are relatively insensitive to Reynolds-number effects. However, the elliptical cylinder represents a class of shapes whose separation points are free to oscillate (due to rounded shoulders); in such cases, Reynolds-number effects can be of primary importance.

A careful review of the existing literature pertaining to the vortex shedding associated with bluff cylinders revealed a continuing controversy in regard to the precise manner in which model yaw influences shedding frequency. Such knowledge is important in the study of the flow noise associated with sonobuoy cables, and also is of great value in predicting the drag of towing cables. The existing experimental data dealing with the particular case of a yawed cylinder lead to contradictory conclusions. Basically, the investigations have been aimed at determining the applicability of the Indepencence Principle described in Schlichting.[16] On yawed cylinders which obey this principle, the characteristics of the crosswise flow are independent of the spanwise flow and depend only on the component of the flow that is perpendicular to the cylinder's axis. If θ is defined as the angle formed by a cylinder's axis and its projection on a plane perpendicular to the flow direction, the normal component of the freestream velocity is $U\cos\theta$. Thus, for a given freestream velocity, the shedding frequency is seen to vary as the cosine of the yaw angle. As a result, the Independence Principle has become synonymous with the so-called "cosine law."

Chiu and Lienhard[11] have analytically investigated the boundary-layer flow over a yawed circular cylinder and found that the cosine law applies if the boundary-layer equations remain valid at the separation point itself. Moreover, they show that when $Re_n \geq 300$ the cosine law can be used to find the pressure-drag coefficient C_{D_n}, whick is based on the force normal to the cylinder and the freestream velocity U. The result is

$$C_{D_n} = C_D \cos^2\theta \tag{3}$$

where C_D is the drag coefficient for zero yaw and the same freestream velocity.

The experimental verification of the preceding results has been a subject of great interest. Chiu's[17] experiments, in the range $2000 \leq Re_n \leq 21000$ and for yaw angles between 0 and 60 deg, appear to confirm the cosine law for shedding frequency. However, similar experiments conducted by Van Atta[18] in the range $50 \leq Re_n \leq 150$ have shown that the shedding frequency decreases very nearly like $\cos\theta$ if $\theta \leq 35$ deg; for all larger angles the decrease is slower than $\cos\theta$. More recently, Smith et al.[19] have obtained data on both the drag and shedding frequency of a yawed circular cylinder in the range $2000 < Re_n < 10000$. Their results show that, although the vortex-shedding frequency closely follows the Independence Principle (cosine law) for all yaw angles considered (0 to 60 deg), the cosine law for the component of pressure drag normal to the cylinder is invalid. However, in view of the manner in which pressure drag depends on wake characteristics, the validity of one of these laws should imply the validity of the other. Since Smith's conclusions regarding drag are well-documented, the preceding remarks still leave the question of the shedding frequency somewhat unsettled. Although the existing Strouhal data appear to have covered the Reynolds-number range below 300 and above 2000, the intermediate range has not been explored. An additional objective of this study has therefore been to obtain such data at a yaw angle which is high enough to reveal any departure or deviation from the Independence Principle.

Experimentation

The facility used for this study was an open-circuit wind tunnel. Immediately behind the 5 ft by 7½ ft (1.5 m by 2.3 m) rectangular inlet was a honeycomb whose elements had an ℓ/d ratio of about 6. Two 20-mesh screens were situated at the entrance to the contraction cone, which had an area ratio of 6¼:1. After passing through a 1½ ft by 4 ft (0.46 m by 1.2 m) test section, the air entered a low-divergence-angle diffuser. The tunnel wind speed could be varied from approximately 6 ft (1.8 m) per sec to 130 ft (40 m) per sec by rotatable radially mounted vanes which were situated between the diffuser exit and blower inlet. The high-ratio gear reduction incorporated in the electric-motor drive for the vanes provided a very fine control over the blockage in the inlet of the constant-speed blower. Estimates of the turbulence in the test section indicated that, at the airspeeds of these experiments (less than 25 ft (7.6 m) per sec), the intensity was under 0.10%.

All of the models used in these experiments were supported by pin vises located at each end. One of the vises was fastened to an arm which rotated in the horizontal plane. The vise at the opposite end was rigidly fastened to a guide rod which was mounted horizontally and parallel to the tunnel wall. In order to suppress or eliminate model vibrations, resilient, vibration-absorbing pads were employed throughout the model-support system. A check of the reliability of the entire apparatus was provided by experiments with unyawed circular cylinders; the Strouhal data from these models showed very close agreement with the most widely accepted data in the literature (as given in a survey by Lienhard[20]).

A highly polished standard brass rod, which had a nominal diameter of 0.094 in. (2.39 mm), was used for the yawed-cylinder experiments. Two noncircular two-dimensional models were studied in the unyawed-cylinder experiments; one of them had an elliptical cross section and the other a square section. The cross section of the elliptical model was identical to the section produced by a vertical cutting plane which is inclined 40 deg to the axis of a horizontal circular cylinder. The streamwise cross section of the circular model at 40-deg yaw was therefore identical to the elliptical model at zero-degrees yaw. The nominal major and minor diameters of an ellipse formed in this manner from a 0.094-in. (2.39-mm) circular diameter are 0.123 and 0.094 in. (3.12 and 2.39 mm), respectively; the corresponding eccentricity is 0.645.

In view of the small size of the elliptical model, the two most critical considerations were orientational referencing and dimensional tolerances. Casting was found to be the only method of fabrication which could meet these requirements. Of the various casting materials considered, "Epolite", which is a high-density epoxy made by the Rezolin Co., was found to possess the most favorable shrinkage characteristics (less than 1%); it was therefore selected as the casting material for both the strut and the end-fins which provided the orientational reference. The actual mean eccentricity of the elliptical cross section in the vicinity of midspan was found to be 0.625, with a corresponding aspect ratio of about 33. A check of surface roughness with a standard surface-roughness comparator indicated that the finish was no higher than a Number 8; i.e., the rms height of the surface irregularities was approximately 8 microin. (0.2 μ).

The square cylinder used in these experiments was fabricated from a piece of ⅛-in. (3.2-mm) steel key stock. The corrosion found on the original surface was completely removed by surface grinding. Moreover, this was found to be the only polishing method which could guarantee uniformity in the model dimensions. After performing the grinding with the aid of a magnetic chuck, the final finishing of the cylinder was made by a light application of fine polishing paper. The width of each face of the completed model, which had an aspect ratio of 28, was found to be 0.122 in. (3.1 mm); the surface finish, based on the roughness comparator, was a Number 16 or better.

A ⅛-in. (3.2-mm) diameter Pitot-static tube (modified-Prandtl design), manufactured by the United Sensor and Control Corporation, was used for the airspeed measurements. Because of the low Reynolds numbers of the present experiments, it was necessary to evaluate the Pitot-static tube for viscous effects on the total or impact pressure. The problem has been investigated both experimentally and theoretically.[22-24] These investigators have shown that the boundary layer can cause the stagnation pressure on a body to differ from the value predicted by ideal-fluid theory. This error in stagnation pressure was accounted for by applying the correction formula proposed by Chambré and Schaaf.[25] When this formula was applied to the Pitot tube used in the present experiments, it was found that the velocity error at the lowest airspeed was not more than 2%. The Pitot-tube readout device was identical, in principle, to the micromanometer developed by Smith and Murphy[26], which was a modified U-tube having a movable cistern or reservoir. Smith presents a very detailed discussion of the errors associated with this instrument, including estimates of their magnitude.

The vortex shedding frequencies generated by the various models were measured by a hot-wire probe positioned in the wake. The signal was fed to a Disa constant-temperature anemometer which was powered by a Kepco regulated-dc power supply. The output of the anemometer was, in turn, fed to the vertical plates of an oscilloscope. By connecting an audio oscillator to the horizontal plates, a Lissajous figure was obtained. By properly interpreting this display, the frequency-match point could be found. This procedure proved satisfactory over the whole frequency range of these

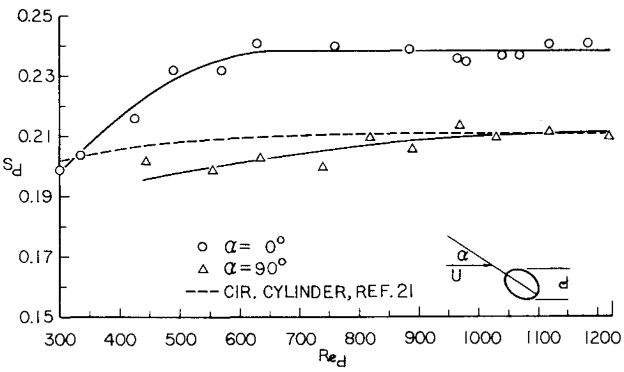

Fig. 1 Variation of Strouhal number with Reynolds number for an elliptical cylinder ($e = 0.625$).

experiments (80–800 Hz) except for those performed with yawed circular cylinders. When making measurements behind yawed circular cylinders, significant frequency modulation in the hot-wire signal caused serious instability in the Lissajous figure and made it impossible to eliminate the multiple loops. A similar situation was encountered by Van Atta[18] who found a modulation of 10%. A plausible explanation of this fact is given by Smith[19] who studied the characteristics of the shear layer for different angles of yaw. Since attempts to employ an electronic counter in measuring the shedding frequencies behind unyawed circular models proved unsuccessful, the only practical method of measurement applicable to the yawed cylinder was direct reading of oscillogram records of the hot-wire output. Frequency averaging was obtained by multiple exposure of the scope-camera oscillograms for each flow velocity.

Discussion of Results

A. General Observations

The variaton of Strouhal number with Reynolds number is shown in Fig. 1. These results, which were based on the dimension d rather than d_0, are shown for the extreme angles of incidence given by $\alpha = 0°$ and $\alpha = 90°$. It has been shown by Roshko[13] and Gerrard[14] that the relative bluffness of different cylindrical bodies, which are symmetrical about the longitudinal (flowwise) axis, can be compared when the Strouhal number and Reynolds number are based on the maximum height of the cross section. An analogous parameter, which applies more generally to shapes at arbitrary angles of incidence, is the dimension d, which is defined as the projected-cylinder height on a plane normal to the flow direction. This dimension is used in describing the data of Fig. 1. As indicated in the preceding references, a change in bluffness is associated with a change in the relative width of the formation region. Since the size (width or length) of the formation region is inversely related to the shedding frequency, a few interesting comments can be made concerning Fig. 1. The bluffness of the ellipse at 90-deg incidence is seen to be very close to that of a circular cylinder, the respective Strouhal numbers being coincident above a Reynolds number of about 1000. The increased scatter at the lower Reynolds numbers is probably attributable to the proximity of wake transition which, for a circular cylinder, lies in the Reynolds number range 150–300; these limits are dependent on the turbulence level of the freestream. The case of zero-degrees incidence, however, shows a definite reduction in bluffness or relative width of the formation region. Also, the Strouhal number is observed to decrease at about the same Reynolds number as the circular cylinder but at a faster rate.

Evidence of the different degrees of bluffness associated with a square cylinder at different angles of incidence is shown in Fig. 2. The characteristic dimension d is again used in forming the Strouhal and Reynolds numbers. From the data it appears that at 29-deg incidence the cylinder is only slightly less bluff than it is at zero-degrees incidence, the Strouhal number levels differing by less than 10%. The influence of the cylinder afterbody on vortex formation is indicated by the comparison with the Strouhal level of a 90-deg wedge at zero incidence. The very close agreement between the results for the wedge and the square cylinder clearly indicates the negligible effect of the cylinder afterbody. The bluffness of the square cylinder at 45-deg incidence is also shown. Although very little data were obtained, it is, nevertheless, apparent that the bluffness is quite similar to that of a plate perpendicular to the flow. Since the data for 29-deg incidence represent the upper limit of the Strouhal number for $Re \geq 10^3$, it can be concluded that the bluffness of the cylinder increases very rapidly as the angle of incidence increases beyond 29 deg.

B. Data Correlations

By combining the zero-incidence data of the elliptical cylinder with similar data obtained from other sources (higher Reynolds numbers), it is possible to illustrate the general dependence of the vortex frequency of an elliptical cylinder on Reynolds number for at least the range of eccentricities between 0.6 and 0.8 and for $500 < Re_{d_0} < 10^5$. For this purpose, the data of various sources have been plotted on a logarithmic grid (Fig. 3). It is observed that all the data above a Reynolds number of 500 lie on the same straight line. For comparison, the best fit to circular data, taken from Roshko[21] is also shown. The separation between the two lines is somewhat suppressed by the logarithmic scale. However, it is seen that the separation diminishes as the Reynolds number increases. The equation of the fitted data is found to be

$$F = 0.27\, Re_{d_0}^{\;0.98} \tag{4}$$

where $F = fd_0^2/\nu$. This relation represents the best fit (within a few percent) over the whole range. At Reynolds numbers below 500, however, the data are well represented by the circular result.

In view of the inverse relationship between cylinder bluffness (relative size of the formation region) and Strouhal number, the geometries of these experiments were analyzed on the basis of the dimension d. For a given cylinder, this dimension accounts for the influence of frontal height on the bluffness at any given angle of incidence; in general, however, the bluffness will also depend on the geometrical details, i.e., the manner in which the flow diverges around the body. To account for the influence of the frontal height d, the S vs α data for both the elliptical and square profiles were recalculated using a Strouhal number defined in terms of this dimension. The

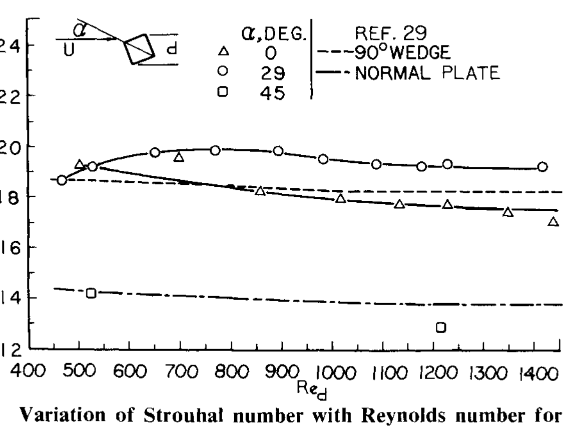

Fig. 2 Variation of Strouhal number with Reynolds number for a square cylinder.

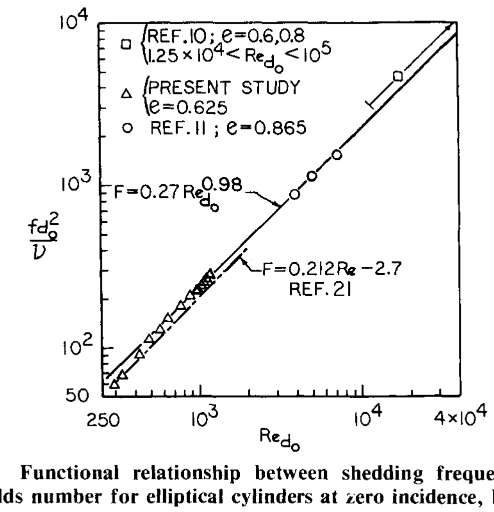

Fig. 3 Functional relationship between shedding frequency and Reynolds number for elliptical cylinders at zero incidence, based on various sources.

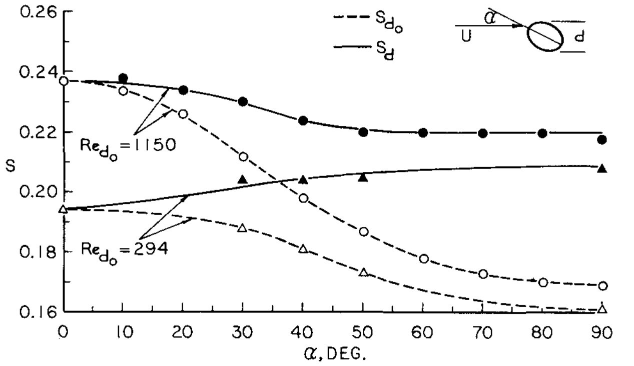

Fig. 4 Influence of projected height on the Strouhal number of an elliptical cylinder.

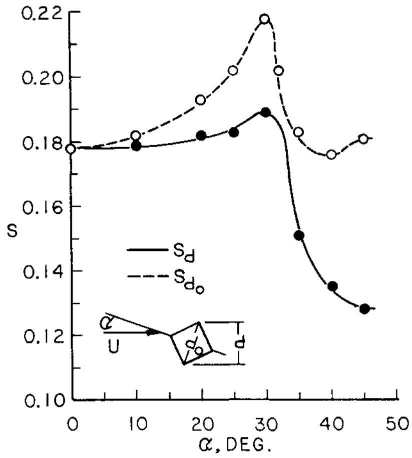

Fig. 5 Influence of projected height on the Strouhal number of a square cylinder, $Re_{d_s} = 1215$.

application of this procedure to the elliptical model yielded the results shown in Fig. 4. Here a comparison is made with the conventional Strouhal-number data. The variation of S_d with angle of incidence is significantly less than that of S_{d_0} for both the Reynolds numbers shown. In fact, for $Re_{d_0} = 1150$ the value of S_d is constant for $50° \leq \alpha \leq 90°$. For the lower Reynolds number, the variation in S_d appears to be very small for angles above about 40 deg.

The shapes of the S_d curves of Fig. 4 can be explained as follows. In the case of the curve for $Re_{d_0} = 1150$, the wake width starts to increase rapidly as α increases from 0 deg; this is indicated by the decrease in S_{d_0}. However, this decrease is not fully accounted for by the increase in the frontal height d; at the lower incidence angles, no proportionality seems to exist between wake width and d. At $\alpha = 40°$, however, it appears that the disparity in the rates of change of the two quantities has diminished. Finally, at $\alpha = 50°$, the wake width seems to have become proportional to d. This implies that any reduction in shedding frequency will be exactly compensated by an increase in d. Hence, the value of S_d will be constant. In the case of the curve for $Re_{d_0} = 294$, the value of d at first increases more rapidly than the wake width, causing a rise in S_d. However, the disparity in the rates of increase of d and wake width diminishes as α increases until finally, at about $\alpha = 75°$, proportionality between d and wake width again exists and S_d remains constant.

In a manner similar to that for the elliptical cylinder, an analysis was made of the S versus α data of the square cylinder. These results are shown in Fig. 5, where the conventional and modified Strouhal numbers are seen plotted for the case of $Re_{d_s} = 1215$. The striking feature of S_d is that it remains nearly constant for angles of incidence up to 30 deg. Beyond this point a sharp decrease is seen to occur. A similar result is

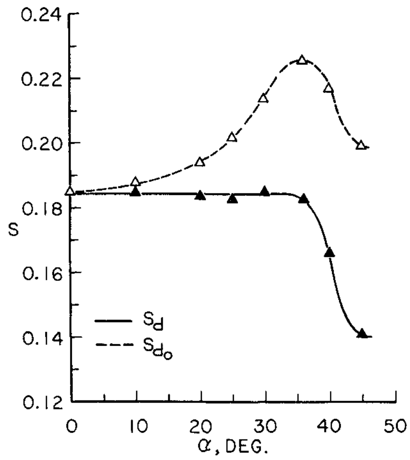

Fig. 6 Influence of projected height on the Strouhal number of a square cylinder $Re_{d_s} = 374$.

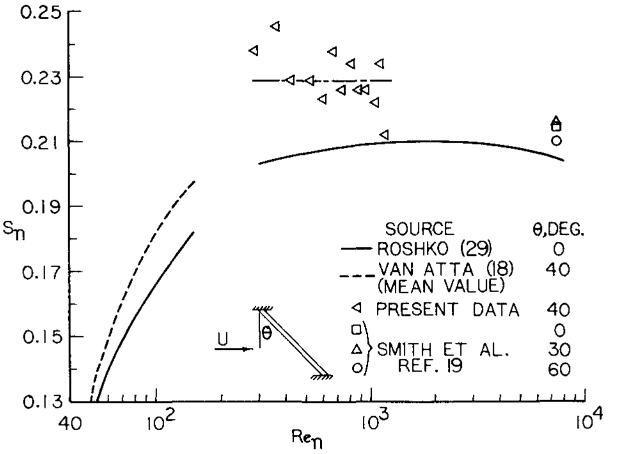

Fig. 7 Variation of Strouhal number with Reynolds number for a yawed circular cylinder.

found for the case of $Re_{d_s} = 374$, given in Fig. 6. Here, however, the peak value of S_{d_0} has shifted to $\alpha = 36°$.

The preceding results clearly indicate that the dependence of wake geometry on cylinder geometry changes suddenly when certain angles of incidence are reached; this fact is undoubtedly related to sudden changes in the streamline pattern. Such changes can, in fact, be seen in the sketches of Parkinson[27], which were drawn from the photographs of Irminger and Nokkentved[28] who studied the flow past a 50-mm square cylinder being drawn through water. From these illustrations, it appears that the sudden decrease in S_d in Figs. 5 and 6 is due to bubble detachment from one of the windward faces (depending on direction of rotation). Once the flow fully detaches itself from the cylinder, changes in orientation would be expected to exert much less influence on the size of the wake. A sudden widening of the wake due to flow detachment would require, from a consideration of vortex formation, a corresponding sudden reduction in shedding frequency.

In order to form more definite conclusions regarding the validity of the Independence Principle, the yawed-circular data obtained in the present experiments were compared to the results obtained in other ranges of Reynolds numbers. In Fig. 7 the Strouhal number (based on the component of freestream velocity normal to the cylinder axis) is plotted against a similarly defined Reynolds number. In the lowest range of Reynolds numbers, the data of Van Atta[18] clearly show a small (but definite) departure from the Independence Principle (solid curve). A mean curve, representing all his data, is found to be approximately 8% above the cosine law. The data obtained at the higher Reynolds numbers of the present experiments show a similar increase of approximately 10%. However, there is some uncertainty in the result due to the rather large scatter in the data. Much of this scatter can probably be attributed to the predominant frequency modulations, which required the use of an alternate data-recording method (described in the preceding). Although this method was reliable, it did not afford extensive time averaging of the velocity fluctuations. Nevertheless, the results definitely support Van Atta's finding that the deviation from the so-called "cosine law" does increase with yaw angle. However, when S_n is converted to S by replacing U_n by U, it is clearly seen that the shedding frequencies are well below those obtained for the elliptical model at the same freestream velocity. This is possibly an indication that a significant portion of the kinetic energy of the freestream is directed into the spanwise flow, and as a result, less kinetic energy is available for the generation of vorticity in the shear layers. It should be noted that the preceding data support the findings of Chiu and Lienhard,[11] who concluded that gross dissimilarity exists in the comparison of the flow past an unyawed elliptical cylinder with the flow past a yawed circular cylinder having the same flowwise cross section. Existing data on yawed circular cylinders at much higher Reynolds numbers are also shown in Fig. 7. Though the general level of this data, due to Smith,[19] is slightly above Roshko's curve, the Strouhal number fails to exhibit any direct dependence on yaw angle. Thus, the Independence Principle, as set forth by Chiu and Lienhard, appears to be uniquely applicable only to this range of Reynolds numbers.

Conclusions

The preceding results reveal that for elliptical cylinders ($0.6<e<0.8$), the variation of Strouhal frequency with Reynolds number can be fitted to a curve analogous to that for circular cylinders; this fit was found to hold over a wide range of Reynolds numbers ($500<\mathrm{Re}<10^5$). The data appear to be well correlated by the formula

$$F=0.27Re_{d_0}^{0.98}$$

These results on elliptical profiles include a range of Reynolds numbers ($300<Re_d<1200$) heretofore unexplored.

The preceding experiments have also shown that a modified Strouhal number, based on the projected height of the model, can be successfully applied at low Reynolds numbers to high-aspect-ratio square cylinders (or similar geometries) to obtain a correlation of shedding frequency with angle of incidence. This correlation has been found to hold fairly well up to angles where complete detachment of the flow occurs on one of the windward faces. Previous investigations in this range of Reynolds numbers appear to be nonexistent.

From the experiments relating to the Independence Principle (cosine law), which covered a range of Reynolds numbers above Van Atta[18] but below that of Chiu and Lienhard[11] and Smith[19], it can be concluded that the results corroborate the findings of Van Atta, viz., that the Strouhal number based on the velocity component normal to the cylinder is approximately 8-10% above the cosine-law prediction at 40-deg yaw.

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